

# Investigating Potential Human Tetrachromacy in Individuals with Tetrachromat Genotypes Using Multispectral Techniques

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## Abstract

This article uses multispectral techniques to investigate color processing in two individuals possessing photopigment genotypes allowing potential human tetrachromacy. In our investigations we measure spectral reflectances from empirically reproduced color sensations of potential tetrachromat observers, and investigate color processing basis functions underlying the observed set of tetrachromat spectra. Our investigations provide new empirical and quantitative methods for estimating trichromat individual's personalized spectral sensitivities, and, as shown in one potential tetrachromat examined, permit estimation of cone response sensitivities for cases that may not conform to the kind of standard dimensional solutions typically associated with trichromat models.

## 1. Introduction

Conventional color technologies — such as digital cameras, computer displays and television systems — are typically based on three display primaries or filters, and are thus inherently constrained by device-dependent metamerism in the reproduction of high quality color images. Constraints on high quality color reproduction that arise from device-dependent metamerism occur when a device's display primaries (or filters) do not provide adequate gamut or intensity ranges to approximate those typical of the human color vision system. When such limitations exist, subtle color appearance variations that may be apparent to a human observer engaged in naturalistic viewing of a scene, can be lost when an image of that scene is reproduced on a display device.

Historically display engineers have addressed device-dependent metamerism by identifying sets of primaries that best accommodate human visual system sensitivities and variations. While three-color-channel systems have long been considered standard, advances in digital display and camera industries now make it both technologically possible and economically feasible to consider extending device primaries beyond the three typically used.

The present article investigates (a) potential image reproduction enhancements made possible by extending the number of display primaries of a color system, and (b) what such advances might imply for not only observers with normal color vision, but also for observers with exceptional color vision owing to color vision photoreceptor processing that potentially confers richer color experience. Here we describe how analyses of potential tetrachromat color vi-

sion can lend insight into effectively addressing the device-dependent metamerism problem, as well as suggest how to generally enhance color representation in digital imaging technology. In addition, results show that studying color vision in potential tetrachromat observers clarifies some color vision features associated with four-photoreceptor color vision phenotypes, which may aid in extending display technologies towards multispectral imaging systems.

## Potential Human Tetrachromacy

Potential tetrachromacy is a recently noted form of human color perception [25], made possible by inheritance of altered X-chromosome-linked photopigment opsin genes responsible for human color vision. Potential tetrachromat individuals carry gene sequences that permit expression of four distinct classes of daylight-sensitive retinal photoreceptors used in the visual processing of environmental color stimuli. Individuals with retinas expressing a tetrachromat potential differ from the majority of people who typically possess genes for three distinct photopigment opsins, and who, as a result, retinally express up to three classes of daylight photosensitive cells. The latter configuration is analogous to that engineered in standard three-primary display devices, and models with three distinct classes of photoreceptors, which, when neurally represented as a trivariant color code, have long been accepted as the standard for normal human color vision and widely considered the basis for typical ranges of normal perceived color appearance.

Genetic variations most often linked to human potential tetrachromacy derive from individual's X-chromosome-linked photopigment opsin genes. Genetic variants, or opsin sequence *alleles*, of standard medium-wavelength and long-wavelength sensitive photoreceptors ("M-cone" and "L-cone", respectively) provide sequence variations that make tetrachromacy possible. Females with two X-chromosomes are capable of retinal expression of four different wavelength-sensitive photoreceptor classes (S-, M-, L- plus a distinct alternative, presumably a variant of the usual L-cone class, often referred to as L'-, or L-prime) making them "retinal tetrachromats", while males with a full complement of X-linked opsin genotype on their single X-chromosome are presumed to express some form of three different wavelength-sensitive photoreceptor classes, conferring such males with color vision trichromacy.

Since the 1980's empirical results have emerged suggesting that retinal tetrachromacy may provide a photopic

response that allows observers to distinguish more colors than that experienced by conventional trichromat color vision. [24–26]. Individuals genotyped as having a potential for tetrachromacy have since been empirically investigated by several different groups, with the general result being that some potential tetrachromats experience color perception, which, in some cases, seems *better than normal*, and is not fully captured by a normal model of trichromat color vision (see [4, 12–14, 18, 19, 27]). Recent empirical investigations compare one specific potential tetrachromat individual (CA) to other control participants and finds empirical results suggesting observer CA, (i) sees more colors under conditions of dim illumination (especially for stimuli having dominant wavelengths coincident with the putative peak of her extra photopigment’s sensitivity response), and (ii) essentially judges some appearances as “colorful” for which a normal trichromat observer’s performance suggests are relatively colorless appearances. [16, 17] In general, potential tetrachromat CA’s results suggest a psychophysical transformation reflecting non-uniform variation compared to a normal observer’s performance – that is, it includes regions of deviation from normal for some color stimuli, while other color space regions showed CA’s processing was indistinguishable from normal processing. [16, 17]

The present article empirically explores the potential perceptual consequences of retinal tetrachromacy through investigations of color perception and the modeling of color processing behaviors using multispectral analysis techniques. Owing to their general utility and efficiency in data inspection, we employ machine learning algorithms to investigate color processing underlying the observed phenomena: That is, estimating optimal color matching functions from tetrachromat behavioral data.

One typical way to address this problem would be to estimate observer’s spectral basis functions based on empirical observations using principal component analysis (PCA) methods. A concern, however, is that due to dimensional orthogonality in the present case, PCA basis function solutions of these data are likely to contain negative coefficients that make physical implementation problematic. An appropriate alternative approach is Non-negative Matrix Factorization (NMF) or Non-negative Tensor Factorization (NTF) which could provide useful non-negative decompositions of these data. Previously NMF and NTF were used to define optimal non-negative representations of spectral colors for data from different sets of stimuli, including the Macbeth ColorChecker (MCC), a Munsell spectral color set and paint spectral sets [1, 5, 20].

As described below we present two distinct investigations of potential tetrachromat processing. First, we collect empirical observations for two observers’ color perception variations of a standardized color stimulus. And, second, we employ the NMF genetic algorithm to calculate approximate basis functions for spectra associated with a tetrachromatic color vision model.

We implement two algorithms to calculate the basis functions. The first algorithm finds basis functions by minimizing mean-squared errors between given spectra and their approximations, and is performed in spectral space.

The second algorithm finds basis functions minimizing color differences when spectra and spectral approximations are converted into the domain of color, thereby providing optimizations in a standard color space. To convert spectra to color we use an existing three channel algorithm [7]. While the first algorithm solves the task in an appropriate way, the second algorithm may over specify its solutions by the assumption of a three-channel color reproduction for tetrachromatic data. Despite this potential limitation, results presented below suggest that it is nevertheless useful to examine three-channel color reproductions from simulated tetrachromatic spectral reflectance. We consider such reproductions as mappings to a three-dimensional space that provides valuable characteristics of calculated basis functions. Three channel reproduction helps to obtain the smoother and better localized basis functions that cannot be achieved with methods working in spectral space. In addition, our experiments with the three-channel system indicate significant modeling improvement in color reproduction compared to spectral optimization, which agrees with existing investigations using three channel systems [5]. Therefore, as detailed below, results obtained suggests that the three-channel color reproduction method in color space is useful for understanding some potential tetrachromat color perception differences explored in the present study.

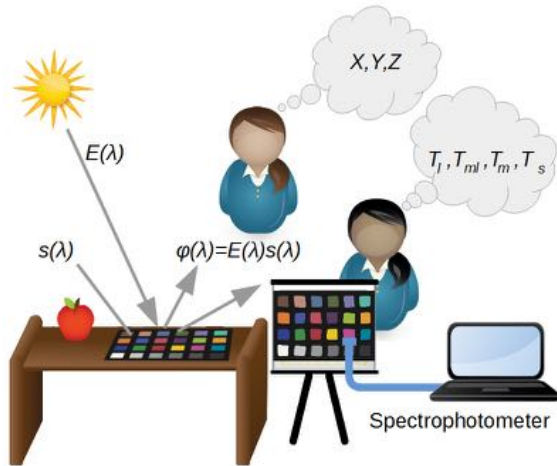
The remainder of the present paper is organized as follows: Color vision models based on empirically observed color perception behaviors are considered in Section 2, empirical methods for collecting tetrachromat spectral reflectance stimuli are described in Section 3, data dimensionality and basis function calculation are explained in Section 4, experimental results are given in Section 5, and results interpretation and discussion are presented in Section 6.

## 2. Two color vision models

Color vision models provide a principled basis for understanding human color perception. In modeling developed here we first consider a trichromat color vision model [11]. We assume that the composition of the spectral power  $E(\lambda)$  of an illuminant and the spectral reflectance  $s(\lambda)$  of an object are sufficient to physically characterize the subjective color of a spectral stimulus  $\varphi(\lambda)$ . The Standard Observer senses these physical stimulus values and integrates them to provide spectral color stimuli using three color matching functions:  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$  and  $\bar{z}(\lambda)$ . The result of the integration can be described as tristimulus values which specify color sensations (Fig. 1) for a trichromat observer as  $X$ ,  $Y$ , and  $Z$ . [31]

Similarly, initial color vision processing in a retinal tetrachromat can be characterized by altering the underlying color matching functions, varying their number, shape and wavelength range. In such cases, tetrachromat processing can be denoted as  $\bar{t}_l(\lambda)$ ,  $\bar{t}_{ml}(\lambda)$ ,  $\bar{t}_m(\lambda)$  and  $\bar{t}_s(\lambda)$  corresponding to the long, middle-long, middle and short sub-range of wavelengths. Tetrastimulus values  $T_l$ ,  $T_{ml}$ ,  $T_m$ , and  $T_s$  thus depend on the integration of the product of four color matching functions and color stimulus spectra (Fig. 1). The tetrastimulus values provide a basis for color sensations arising from tetrachromat retinae. Thus, the two color vi-

sion models presented provide formal descriptions for color sensation variations occurring in a standard trichromat observer compared to a tetrachromat observer.



**Figure 1.** Two color vision models based on tristimulus and tetrastimulus values.

An important component for understanding tetrachromat color perception is the accurate estimation of tetrachromat color matching functions, or chromatic response functions that arise from the initial limiting constraints found in cone spectral sensitivity variants [8,21]. Previous research suggests how cone fundamentals can be simulated for a potential tetrachromat [19]. By comparison, here we simulate cone fundamentals using a machine learning algorithm which provides approximations of the color matching functions underlying observed color perception behavior, and provides a new engineering-based approach for studying the perceptual consequences of retinal tetrachromat color vision processing.

### 3. Color Reproduction Experiment

The analytic methods developed in Section 4 make use of personalized color perception variation that is assessed through empirically observed color reproduction.

#### 3.1 Participants

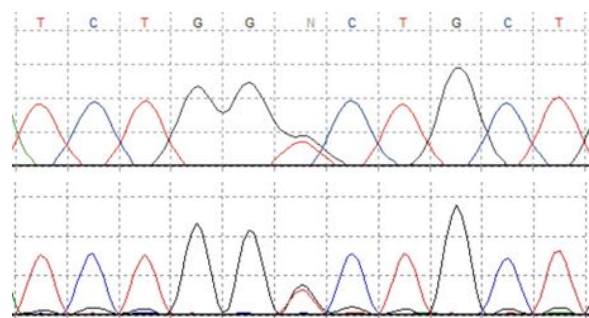
As shown in previous photopigment opsin genotyping research molecular genetics research has determined that genotypes involving more than three normal photopigment opsin variants are not uncommon, [3,4,8,13,14,19,21,27] and it is thought that mechanisms governing expression of such photopigment opsin genes does not rule out the possibility that an individual will express more than three classes of retinal photopigments. The aim of much of the research into potential human tetrachromacy has been to discover (a) how the possession of extra photopigment opsin genes may alter perceptual processing of color, and (b) what the X-chromosome linked features of the L-cone and M-cone opsin genes implies for potential human tetrachromacy and gender-linked color vision processing differences. [15]

Here we describe preliminary investigations of color perception in two opsin genotyped observers. The first individual is female participant “CA” who is assessed as having superior color perception on standardized color vision tests,

and who was previously studied and genetically confirmed as possessing a potential tetrachromat genotype. [16,17] The second participant is female participant “LG,” also assessed as having superior color perception, but whose photopigment opsin genotype was unknown prior to the color perception assessment presented here, and who, based on familial color vision information, was considered to be appropriate as a trichromat control participant prior to opsin genotyping. Surprisingly, however, participant “LG” once genetically assessed was found to possess a potential tetrachromat genotype. [17] Thus, in the present investigations we present and compare two participants with genotypes for potential retinal tetrachromacy.

In addition to being genotyped as potential tetrachromats, CA and LG both have (i) varying experiences in art education, training and practice; and (ii) have portfolios as oil painters that include art works on diverse ranges of naturalistic objects and scenes, and have sold and/or exhibited their paintings for profit. Thus, both participants are confirmed as having a genetic potential for color vision tetrachromacy, have considerable color perception learning experience through art education and practice, and have excellent art skills.

As previously reported [16,17], CA and LG volunteered for participation in a series of investigations involving color perception for which participants’ opsin DNA sequences were genotyped using a novel implementation of polymerase chain-reaction (PCR) methods. [2,13,28] All genotyping was performed with participants’ informed written consent using procedures that adhere to protocols compliant with the world medical association declaration of Helsinki ethical principles for research involving human subjects, and were approved by the ethical review board of the University of California, Irvine. Figure 2 shows the L-opsin Exon 3 Ser-180-Ala genetic sequence excerpts for the two participants discussed here.



**Figure 2.** Genetic sequence detail of the L-opsin gene for two different potential tetrachromat participants. Top panel depicts an excerpt of the L-opsin gene sequence for CA, and the bottom panel shows the corresponding sequence for LG. Curved peaks depict the intensity of the nucleotides (dNTPs) observed in the DNA. For both participants column 6 of the sequences show L-opsin Exon 3 codon 180 loci where two allelic variants – corresponding to serine and alanine – are apparent.

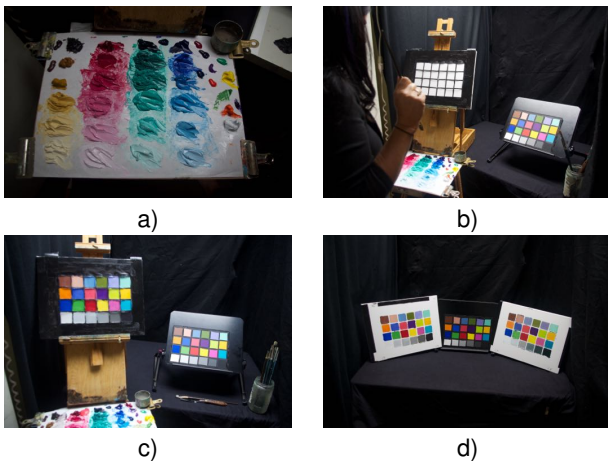
### 3.2 Methods

**Subjects and Design.** Phase 1 involved 1-2 hours of testing to assess color vision using standardized methods.

Phase 2 involved collecting self-paced painted color samples from participants. Total approximate duration of experimental participation during a single session was between 4 and 4.5 hours. All behavioral investigations were performed with participants informed written consent, as approved by the ethical review board of the University of California, Irvine.

Phase 1: Participants were assessed using standardized procedures and some novel analysis approaches. Diagnostics for color deficiency used were Ishihara Pseudo-isochromatic Plates, the Farnsworth-Munsell 100-hue Test, and the O.S.C.A.R. flicker photometric test and the Neitz anomaloscope matching task. Both participants were classified on all tests as having normal color vision.

Phase 2: Following standardized color testing, both participants were assessed in a color reproduction experiment to obtain (i) observer-specific reproductions of the Munsell Color Checker standard stimulus [22], for evaluating observer's individual variation in color perception; and (ii) quantification of personalized color perception variations relative to a known standard observer model, for the purposes of designing color vision models described in Section 4 below.



**Figure 3.** Color Reproduction Experiment. Two research participants reproduced the 24 color appearances of the Munsell Color Checker. Panels shown depict a set-up that was identical across both participants' experimental sessions. Both artists a) started with identical paint pigment palettes. b) Reproduced the 24 MCC squares under the same illuminant (D65), at the same viewing distance, and in the same physical viewing booth set up. c) Were permitted as much time as needed to reproduce the 24 MCC colors, and finished when they were satisfied that their painted version faithfully reproduced the color appearances of the MCC. d) Completed the MCC reproductions (shown beside the MCC stimulus) in separate self-paced sessions in under 40 minutes.

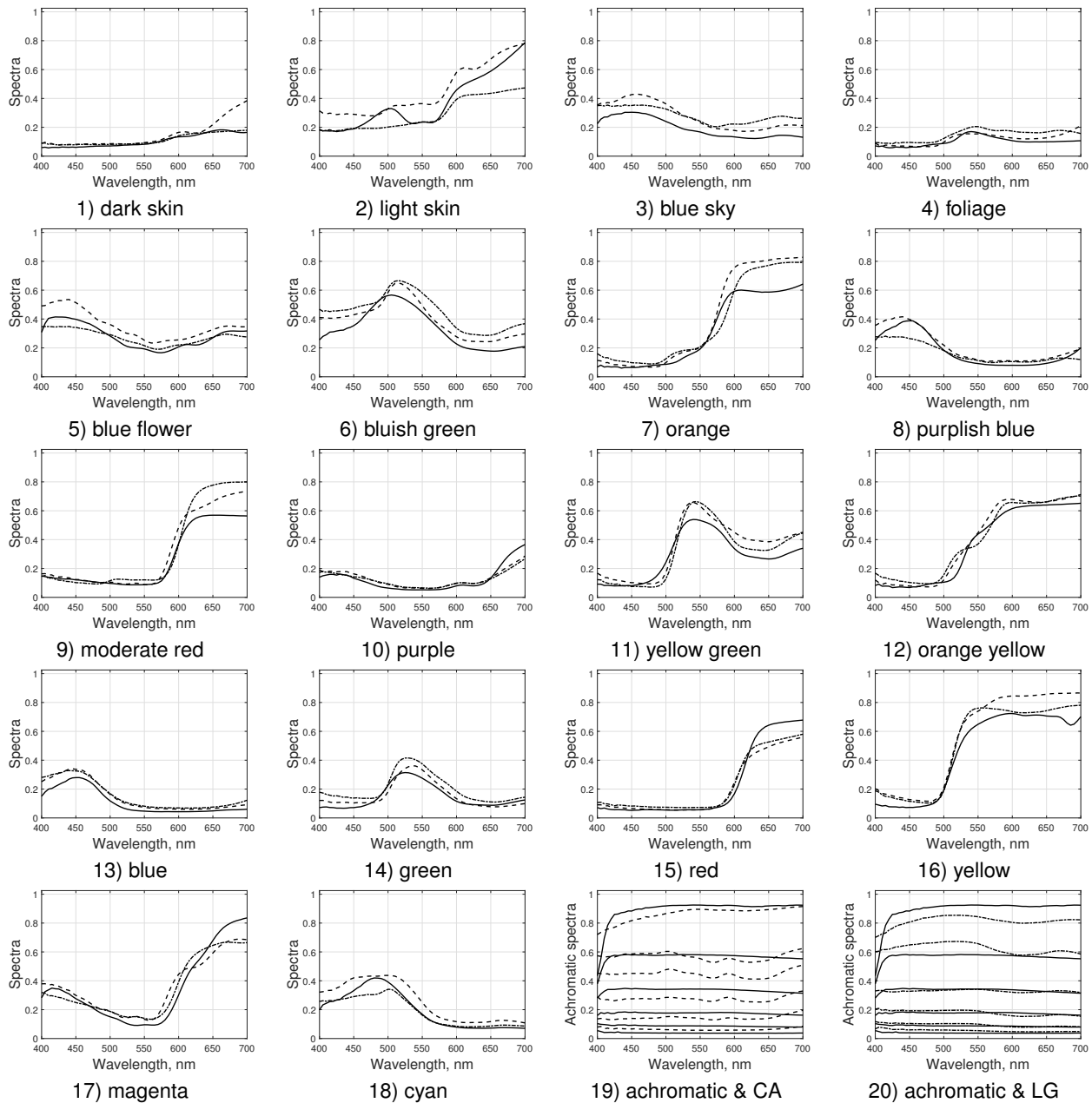
Fig. 3 depicts the empirical setting, pigment palettes used, and reproductions produced by participants CA and LG. At the start of each session a palette of agreed-upon pigments was created and two identical copies of the palette were made for use in the individual experimental sessions. The palettes were comprised of 18 blobs of unique oil pigments. In addition, the palette included 16 additive mixtures of white pigment blended with one of four possible

primaries, where the four were from among the 18 pigments employed. Thus, the palette consisted of 18 unique pigment blobs, and 16 pre-mixed blobs of pigment-plus-white. (see Fig. 3a). As shown in Fig. 3b, the viewing booth set up was illuminated from above by a standardized lightsource approximating a D65 illuminant using 4 lightbulbs in a fixture designed for optimal light distribution, and standardized for industrial proofing and color control [10]. Lamps were warmed-up 1-hour prior to start of the experimental session and participants were adapted to the environment for a minimum of 30 minutes. During the session participants were permitted free viewing of the stimuli, the palette and a canvas (configured with a dark MCC stimulus surround overlay that outlined 24 segments, of 4 cm. square, of blank canvas to be painted), which were viewed at estimated distances of ~75 cm., ~40 cm., ~60 cm., respectively. The two participants were run under two illuminant conditions: First, for a D65 illuminant and, second, adapted under a filtered chromatic illuminant (only results from the D65 illuminant condition are reported here). Fig. 3c depicts one participant's completed reproduction set shown in the experimental setup. Fig. 3d depicts both participant's completed reproduction sets adjacent to the MCC.

### Color reproduction task rationale

Using a color reproduction task for assessing individual variation in color perception is, admittedly, a nonstandard empirical design. Nevertheless, we chose it based on the pragmatic rationale that both participants (a) have substantial color training and expertise, and (b) are accomplished painters who demonstrate high-level skill at reproducing their color experiences using paints on canvas. The task assumes that to the degree that the observers could accurately reproduce the appearance of color stimuli with pigments, comparing individual differences of participants relative to a standard observer model could provide useful information on the color perception variation across the individual observers. The instructions to participants were to "take as much time as needed to *exactly duplicate* the 24 colors shown [the MCC stimulus] using the pigment palette provided, paying the utmost attention to color reproduction accuracy" ... "you may return to adjust any one of the color patches at time," before signaling to the experimenter that you have either (1) achieved an identical color match between the samples and reproductions, (2) achieved an identical, or satisfactory, color match between some samples, but not all samples (please specify which), or (3) were unable to achieve a satisfactory color match between the samples and their reproductions using the pigment palette provided.

Here we use reflectance measures of the Macbeth ColorChecker stimuli and measures of the empirically painted reproductions of the MCC that are independently generated by each potential tetrachromat participant. Paintings of MCC color patches are made under laboratory conditions at constant illumination. The observed colors of MCC and painted patches are a composition of spectral reflectance and illuminant spectral power distribution. We assume that if a trichromat model of color appearance is appropriate, then, for a given potential tetrachromat observer, if they



**Figure 4.** MCC spectra for chromatic stimuli (panels 1-18) and for composite graphs depicting MCC curves for 6 achromatic stimuli with the corresponding participant curves (panels 19-20). Panel enumeration corresponds to the series shown in Fig. 5, such that MCC row 1 colors from left to right correspond to panels 1-6 shown here, MCC row 2 colors correspond to panels 7-12 shown, and MCC row 3 colors correspond to panels 13-18 shown. Panels 1-18 depict spectra for MCC (solid line), CA (dash line) and LG (dash dotted line). Panel 19 shows CA's spectra (dashed) compared to 6 achromatic MCC spectra. Panel 20 shows LG's spectra (dot-dashed) compared to 6 achromatic MCC spectra. Six achromatic stimuli are depicted as Fig. 5's achromatic series shown. Color nomenclature from [23]

empirically report that a painted patch visually matches an MCC patch, then the measured spectral reflectance of painted patches should have the same  $X, Y, Z$  tristimulus values as the spectral reflectance of MCC. Moreover, we assume that if we observe that measured MCC and painted patch colors that have the same  $X, Y, Z$  tristimulus values appear different to a given potential tetrachromat observer, then we suppose that such differences in appearance primarily depend on features of observer perceptual processing that differ from the tristimulus model (however, see further discussion below). Thus, regardless of which situation obtains, individuals' underlying observer models can be investigated using analyses of measured spectral reflectances of painted patches compared to reflectances of the MCC standard.

For modeling purposes we use spectral reflectance measures of the 24 painted reproductions of the MCC stimuli to compare with the reflectance measures of the standard MCC stimulus, which provides us with an informative database for analyzing each participant's individual color vision mechanisms. That is, because every painted color patch has particular spectral reflectance, we posit that to the degree spectral reflectance measures of a painted patch differs from spectral reflectance measures  $s(\lambda)$  of a corresponding MCC patch, that difference relates to each participant's individual metamer color sensations.

Of relevance here are remarks of Foster et al.: "...metamerism occurs when light with different spectra appear the same to the eye/sensor system. Metamerisms arise because the number of degrees of freedom in a sensor system — three for the cone receptors in the normal human eye or filters of a typical RGB camera — is smaller than the degrees of freedom needed to accurately represent light spectra." [9] And, in the present investigations the dimension of a sensor space could be 3 or 4, and the dimension of a spectral space is generally higher (e.g., here it equals 61, as defined in Section 5.1). Thus, here mappings from color (colored patch) to spectra (simulated spectra) are one-to-many, and observed  $X, Y, Z$  correspondences between simulated spectra and MCC spectra can be effected by at least two different sources of influence.

For example, influences may arise from, (1) perceptual experience variations attributable to tetrachromacy that provide evidence (in the form of disrupted metamer appearance equivalence relations between classically modelled pairs with equivalent tristimulus values) against a tristimulus model as appropriate for describing tetrachromat color processing. Or, (2) cases where identical appearances arising from tristimulus metamers (colors are the same but reflectance spectra are different) turn out by chance (see [9]) to be similarly metameric for a tetrachromat, thereby incorrectly providing support for a tristimulus model of tetrachromat color processing when a trichromat model is actually either inappropriate or only locally appropriate (following a possible dimensional degeneracy in some spectral region).

Based on the foregoing, we generally assume that a spectrum of a painted sample can be described as a "simulated spectrum" for that observer. An observer's set of

simulated spectra can be represented by some number of basis function that accurately describes the entire set of reproduced spectra. An Ocean Optics USB 2000 spectrometer (Fig. 1) was used to measure each participant's 24 color reproductions which were compared to the known measures of the MacBeth Color Checker Classic. [22]

MCC stimulus measurements are shown in Fig. 4, and Fig. 5 shows approximations of color appearances found in the MCC standard stimulus. [23] Inspection of the measurements reveal that the spectra corresponding to yellow/green (Fig. 5's panel 11) and some orange/yellow colors (Fig. 5's panels 7, 12 and 16) are less uniform for CA and LG relative to measures for those MCC spectra. The manner with which they were found to vary from the MCC standard qualitatively suggests that participant CA and LG render the noted stimuli as more saturated as reproduced colors. The achromatic spectra for CA clearly indicate that CA has enhanced sensitivity for dark gray shades. Her spectral values for patches 21–24 are higher than MCC spectra. The LG achromatic spectra are close to MCC spectra.

The qualitative differences seen in the reflectance spectra comparisons shown by Fig. 4's panels are further investigated in Sections 4 and 5 below.



Figure 5. Approximate colors appearances simulating the MCC standard stimulus.

## 4. Data dimensionality and basis functions

To provide a more objective assessment of Fig. 4's reflectance spectra comparisons, non-negative matrix factorization (NMF) is used to define the optimal non-negative representation of color spectra for each participant's simulated spectra of MCC. The NMF method represents spectra by a few non-negative basis functions. NMF determines non-negative factors  $W$  and  $H$  using non-negative factorization of the given spectral data  $V$  as follows:  $V = WH$ . The matrix size is defined as:  $V \in \mathbb{R}^{d \times n}$ ,  $W \in \mathbb{R}^{d \times l}$  and  $H \in \mathbb{R}^{l \times n}$ , where  $d$  is the number of wavelengths,  $n$  is the number of spectral colors, and  $l$  is the number of basis functions. Thus, the columns of  $W$  are the basis functions, and  $H$  is a matrix of weights, that minimize the error of the approximation.

### 4.1 Data dimensionality

We define the data dimension for MCC and simulated MCC spectra. We use an ISOMAP technique which finds a low-dimensional embedding from a high-dimensional observation. For estimating data dimensionality ISOMAP is superior to methods such as Principle Component Analysis (PCA) and Multi-Dimensional Scaling (MDS) [29]. To define data dimensionality ISOMAP analyzes the data matrix  $V$  and calculates a residual variance against the number of embedded components.

The ISOMAP residual variance is defined:

$$1 - R^2(D_G, D_Y), \quad (1)$$

where  $R$  is the correlation coefficient taken over all elements of  $D_G$  and  $D_Y$ ,  $D_Y$  is the matrix of Euclidean distances in the low dimensional subspace recovered by ISOMAP and  $D_G$  is the graph distance matrix of input data  $V$ .

#### 4.2 Basis functions for simulated spectra

Elsewhere we propose two NMF approaches minimizing spectral differences and color differences, both of which use a genetic algorithm (GA). [5] Bochkov et al.'s first approach used spectral differences which provided results similar to standard NMF. Their second approach, however, was based on color differences and produced much more desirable results compared to the first approach based on spectral differences. In the latter case, the basis functions are more orthogonal and bell-shaped because their tails decay fast.

As reported previously, [5] the approach based on spectral differences utilizes an optimization method with a non-negativity constraint. The non-negative matrix factorization solves the following objective function in spectral space:

$$\min_{W, H} \sum_{i=1}^n \|v_i - Wh_i\|^2 \quad \text{subject to } W, h_i \geq 0, \quad (2)$$

where  $v_i$  and  $h_i$  are the columns of the matrices  $V$  and  $H$ , respectively, and  $W$  and  $h_i$  have all non-negative elements.

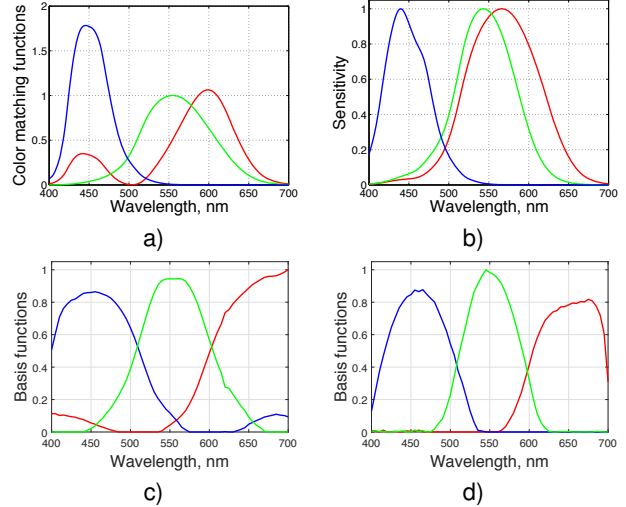
The NMF method gives surprisingly good results when comparing MCC spectra with a model based on Standard Observer color matching functions [31] and cone fundamentals [32] (Fig. 6). Although basis functions are not color matching functions, they are positive and resemble color matching functions in shape, and they serve as limiting conditions for color matching, as well as capture operating characteristics and information about receptor class functional responses to spectral wavelengths (Fig. 6c).

By comparison, the approach based on color differences adopts an objective function in color space. That is:

$$\min_{W, H} \sum_{i=1}^n \|f(v_i) - f(Wh_i)\|^2 \quad \text{subject to } W, h_i \geq 0, \quad (3)$$

where  $f()$  is a spectrum-to-color conversion function [7]. Then  $\|\Delta E_{abi}^*\|^2 = \|f(v_i) - f(Wh_i)\|^2$ , where  $\Delta E_{ab}^*$  is a CIELAB color difference:  $\Delta E_{ab}^* = \sqrt{(L_v^* - L_{Wh}^*)^2 + (a_v^* - a_{Wh}^*)^2 + (b_v^* - b_{Wh}^*)^2}$ . We denote  $[L_v^* \ a_v^* \ b_v^*]^T = f(v)$  and  $[L_{Wh}^* \ a_{Wh}^* \ b_{Wh}^*]^T = f(Wh)$ . [6]

Fig. 6d shows that the basis functions optimized in color space are well-formed and better localized than those ones optimized in spectral space (Fig. 6c). The color space basis functions are much more accurate than in spectral space reducing the approximation error of spectral colors by a factor of 6 for MCC colors [5].



**Figure 6.** Spectral curves. a) CIE 1931 color matching functions of Standard Observer. [31] b) The Smith and Pokorny (1975) cone fundamentals. [32] c) Normalized basis functions calculated using the MCC color spectra and optimized in spectral space. d) The normalized basis functions calculated using the MCC color spectra and optimized in color space.

## 5. Simulation Investigations

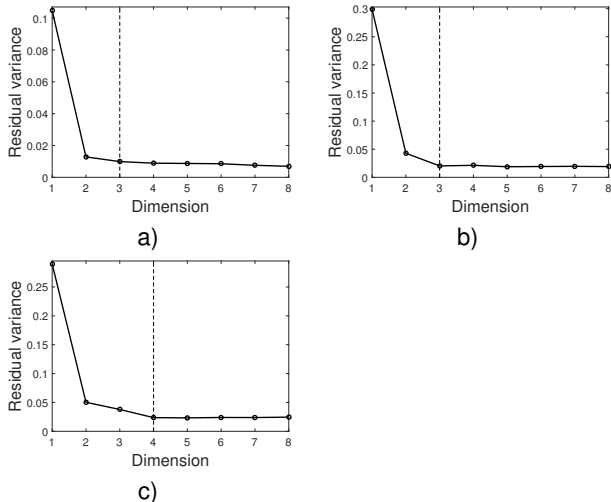
Investigations were conducted to obtain empirically-derived basis functions from sets of simulated tetrachromat spectra.

### 5.1 Basis functions

Three distinct sets of basis functions were calculated for MCC and two observers (LG and CA) from simulated spectra using optimization in two spaces: spectral space and color space (Eq. 2 and Eq. 3). For simplicity, a subset of available wavelengths were employed in the range [400,700] nm, producing sets of 61 values taken at 5 nm intervals. We investigate scenarios that permit variation in the number of basis functions in a solution. The number of the MCC spectral colors was constrained to the 24 provided in the standard stimulus (Fig. 5). Data from two potential tetrachromats described earlier, LG and CA, were measured from each participant's painted color patches using a spectrophotometer (Fig. 1). Thus, we have three spectral sets: MCC, MCC-LG and MCC-CA.

We first aimed to define the data dimension for each data set. For that we use ISOMAP and plot a residual variance (Eq. 1) versus dimension (Fig. 7). The heuristic suggests to select the point where the residual variance values asymptote. ISOMAP was run with neighborhood parameter 4. ISOMAP shows that the solution obtained for the MCC-CA spectral set is higher than the others and equals a 4-dimensional solution. By comparison, the dimensional solutions observed for both MCC and MCC-LG show residual variance appears to asymptote at 3 dimensions.

In addition, data dimensionality is taken into account in our NMF algorithm. The NMF algorithm uses Singular Value Decomposition (SVD) for initialization [30]. The SVD method uses  $p + 1$  leading components of SVD decomposition, where  $p$  leading components contain less than 90%



**Figure 7.** Data dimension shown by a vertical dash line. a) MCC spectra. b) LG simulated spectra. c) CA simulated spectra.

of a total variance. The number  $p + 1$  defines the rank of factorization. For two leading components of the MCC-LG set and three leading components of MCC and MCC-CA sets the 90% threshold is not exceeded. Thus, a three basis function decomposition for MCC-LG data and four basis function decompositions for MCC and MCC-CA data are observed. For MCC-LG data increasing the variance-accounted-for threshold to 93% makes a four basis function decomposition possible as well.

While our ISOMAP and SVD analyses indicate that MCC’s dimensionality is either 3 (in the ISOMAP case) or 4 (in the SVD case), for reasons elaborated below we believe the correct interpretation of these results is that a three dimensional model is appropriate for both MCC and LG spectral data sets, and that a four dimensional model is required to characterize the CA spectral data.

Note, while we interpret the above mentioned results as suggesting MCC and MCC-LG spectral data sets are three-dimensional, we still evaluate all datasets using both 3 and 4 basis function factorization for the purposes of comparatively assess differences and similarities that may arise due to factorization with a higher rank for the tested sets.

Moreover, we compute a four basis function decomposition for LG because her photopigment opsin genotype confers a potential for expressing four distinct classes of retinal photopigments, and suggests that four distinct signals could arise in LG at the retina level. To cross-check validity of our procedures, we also compute a four basis function decomposition for the MCC because we expect to verify that four-bases do not work equally well for the MCC case, which was developed assuming a trivariant model of color processing. Carrying out decompositions using 4 basis functions for all three test cases allows us to show the procedures we use distinguish between the cases considered, and permits a means of evaluating the dimensional assumptions used in the design of the MCC and inherent in the standard normal trichromat observer model.

For MCC spectra, basis functions were calculated

(Fig. 6). Calculated basis functions were normalized twice. NMF can produce ambiguous solutions with respect to factors  $W$  and  $H$ . To avoid this problem, we divided all elements in each column of  $W$  by their sum. After that we normalized all basis functions to make their total maximum equal unity.

We compare the basis MCC functions with color matching and cone curves. The correspondence between the color matching functions and basis functions optimized in spectral space and color space is rather good (Fig. 6). Except for the L-cone estimates, the correspondences to cone fundamentals are also good. Most of basis functions are bell-shaped like color matching or cone curves. The MCC basis functions based on optimization in a color space, are for the most part, more “accurate” than those optimized in a spectral space (Fig. 6 and Table 1). Thus, while Table 1’s data simply provide *rule-of-thumb* indices of correspondences between estimated basis functions and psychophysical measures in spectral and color spaces, the table shows qualitatively, at least, that *color space* basis functions are better correlated with matching curves and cone fundamentals — a result that makes sense given that the MCC stimulus is derived from a color appearance space model and not a spectral space model. In addition, visual inspection suggests they correspond more closely in shape and peak modes for the middle and short wavelength curves. The long wavelength curve, by comparison, does not strongly resemble the empirical L-cone curve, and provides only an approximate representation of the basis underlying the range of longer wavelengths. Therefore correlation coefficients are not given for cone fundamentals for long wavelengths.

**Table 1. Correlation coefficients between color matching functions and cone fundamentals and basis functions optimized in spectral and color spaces.**

Color matching functions			
Wavelengths	long	medium	short
Spectral space	0.10	0.99	0.83
Color space	0.16	0.95	0.90
Cone fundamentals			
Wavelengths	long	medium	short
Spectral space	—	0.97	0.89
Color space	—	0.98	0.91

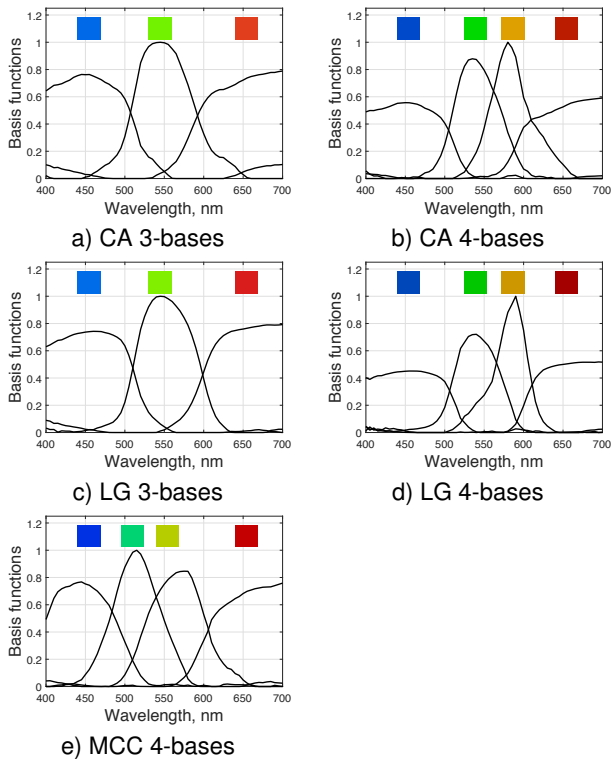
In general the correspondence between optimization results and color matching functions is good for the middle range of wavelengths and for the short wavelengths and not accurate for the long wavelengths. This suggests that if, as modeled in the literature [23], the optimization for the MCC set gives rise to a good trichromat result, in the sense that it captures three bases resembling those used in the original MCC model. Next we extend our measurements, expecting a similar generalization when used as optimization procedures for potential tetrachromat spectral sets, and investigate the derived bases that best capture the potential tetrachromat participant’s data.

In the next investigations we will employ the simulated spectra measured using painted color patches by potential



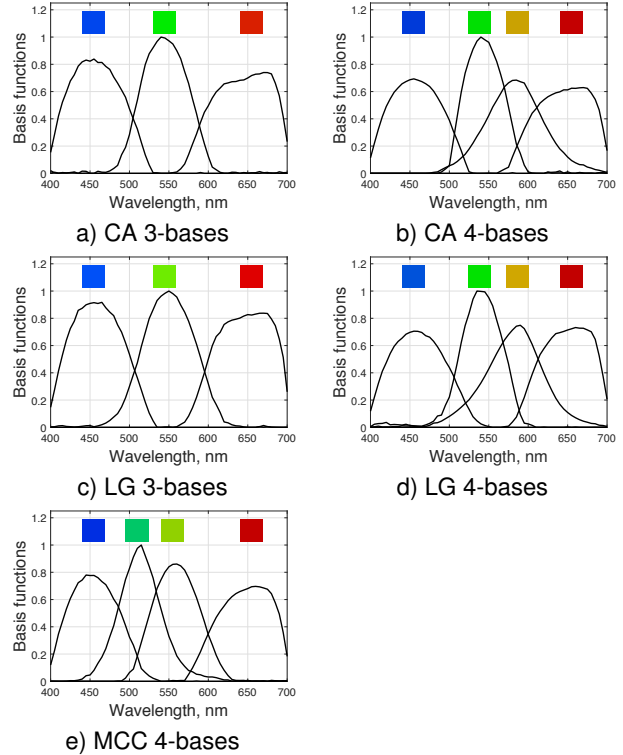
tetrachromats (Fig. 8 and Fig. 9), where spectral curves are depicted as dashed (CA) or dot-dashed (LG) lines. To give a general representation about color corresponding to each curve we depict a color patch to give a qualitative approximate of each curve's peak.

Consistent with earlier observations, results obtained in spectral space are not as impressive as those obtained using color space optimization (Fig. 8 and Fig. 9). Moreover, the basis functions obtained using spectral optimization and corresponding to red, green and blue colors are less orthogonal. However, the curves behave similar to color space curves.



**Figure 8.** Optimization in spectral space. The normalized basis functions calculated using the MCC spectra and the simulated tetrachromatic spectra. a) Three basis functions (CA). b) Four basis functions (CA). c) Three basis functions (LG). d) Four basis functions (LG). e) Four basis functions (MCC).

The optimization results in color space for three basis functions (Fig. 9a and c) are rather close to the MCC test (Fig. 6d). To test whether the MCC was also characterizable with greater than three-dimensions, we conducted analogous investigations assuming that the MCC was four-dimensional. The results independently distinguished four similar tetrachromat basis functions that CA and LG share in common (Fig. 9b and d), which differ from the four basis functions derived for the MCC (Fig. 9e). In this case there is a correspondence between the MCC, CA and LG solutions only for curves similar to those of a trichromat model, while for the cases of the fourth bases obtained, the MCC's solution differs from those found for our two participants' fourth basis functions. For participants' simulated spectra the fourth curve locates in the subrange in the "yellowish"



**Figure 9.** Optimization in color space. Normalized basis functions calculated using the MCC spectra and two sets of simulated tetrachromatic spectra. a) Three basis functions (CA). b) Four basis functions (CA). c) Three basis functions (LG). d) Four basis functions (LG). e) Four basis functions (MCC).

region of the spectrum, while the fourth curve of the MCC spectra locates the fourth basis in the spectral subrange corresponding to bluish green (or cyan) color appearances. As mentioned, the four basis functions for CA and LG are similar (Fig. 9b and d)), and, interestingly, coincide with the approximate spectral location where the sensitivity peak of the putative fourth photopigment class is predicted to occur given the L-opsin codon-180 heterozygosity that both observers possess [16, 17].

### Comparing ISOMAP and NMF solutions

Given the interpretable basis function results found for both potential tetrachromats assessed, it seems important to elaborate on the ISOMAP finding that the data of potential tetrachromat LG exhibits a three-dimensional — and not a four-dimensional — solution. In this regard it can be noted that, following a perceptual learning rationale, it is not unexpected that despite LG's potential tetrachromat genotype, her spectral data set is modeled by only 3-dimensions in conjunction with 3- or 4-basis functions. That is, in view of what has been suggested elsewhere concerning the need for perceptual learning in order to realize functional (i.e., dimensional) tetrachromacy [17], it may not be a surprise that part-time artist LG — who although a skilled artist does not have the extensive longitudinal exposure and continued daily immersion using color appearance judgments

compared to full-time professional artist CA — shows evidence of an extra basis that actually concurs with her genotype, but who does not show as substantial of dimensional deviation (based on the ISOMAP results) as CA who previously was found to empirically exhibit deviations from normal trichromat color vision compatible with dimensionally richer color experience. This pattern of results for LG and CA is consistent with widely-observed finding that individuals with the genetic potential for tetrachromacy frequently do not empirically exhibit functional tetrachromacy. While the idea is currently conjecture, presumably this is due to the plausible scenario that in addition to a genetic potential, exceptional training and perceptual learning is needed to realize functional human tetrachromacy, and, in the absence of a lifetime of color training and practice, many individuals with tetrachromat genotypes, who also express four distinct retinal photopigment classes, may still be found to exhibit normal three-dimensional color vision.

For these reasons, we interpret this above mentioned pattern of results to suggest that potential tetrachromat CA most likely has four different photoreceptors with response sensitivity characteristics that are approximated by the calculated curves (Fig. 9b). The fourth basis function peaks between the long-wavelength and middle-wavelength basis functions while the remaining functions are close to the basis functions of a trichromat. This is in agreement with what is known from participant CA's photopigment opsin genotype [16,17], and is consistent with the simulated cone fundamental results found in other investigations of potential tetrachromats [19]. The simulated cone fundamentals derived here for potential tetrachromat CA contain a fourth spectral sensitivity curve located between spectral curves of M-cone and L-cone while the others curves correspond to those seen in accepted standard models, such as the Smith-Pokorny cone fundamentals for S-, M- and L-cone classes. Previous empirical results for CA also support the existence of an additional factor in the spectra region corresponding to the four basis function here, that was found to influence (and differentiate from trichromat control participants) CA's perception of color in minimum-motion isoluminance settings. [16,17]

Note, these findings agree with earlier results for CA from minimum-motion at isoluminance tasks. [16,17] That is, comparisons of individual's isoluminance settings found CA's results to differ markedly in some regions of color space compared to normal trichromat controls. Jameson and colleagues [16,17] report that the comparative sensitivity found in CA's settings suggest she is expressing a fourth cone class (presumed to be a long-wavelength sensitive cone class variant, based on her genotype) which is contributing to cues used to establish isoluminance equilibria, in addition to the usual signal contributions from M-, L- and S-cones that normal trichromats possess. Moreover, the present basis function estimation results agree with Jameson and colleagues [16,17] findings that CA's greatest deviations from normal trichromat controls are found for stimuli comprised of dominant wavelengths components from regions between the standard normal M- and L-cone peaks — or, the spectral region where her putative fourth

cone class is expected to be most sensitive based on her X-linked opsin genotype. Both sets of results for CA suggest that she possesses a potential tetrachromat genotype that when phenotypically expressed permits richer color experience compared to trichromat controls. [13,14]

## 6. Discussion

Here we describe preliminary results using new modeling and analysis procedures for investigating spectral reflectance measurements of MacBeth Color Checker stimuli compared to measures of painted MCC color reproductions obtained from two potential tetrachromat artists. The sets of reflectance measures reveal suggestive interpretable differences between simulated spectra and MCC spectra in different subranges of wavelengths. Measurement of achromatic spectra sets additionally indicate that, consistent with existing results, one potential tetrachromat (CA) has enhanced sensitivity to chromatic content inherent in otherwise dark achromatic, or gray, appearances.

Based on reflectance measure data from empirical color reproduction tasks we derived individual color processing basis functions for two potential tetrachromats (CA and LG) by assessing NMF results while varying the number of basis functions obtained by solutions. Interestingly, the NMF analysis findings from our four basis function factorization procedures show results consistent with existing psychophysical and opsin genetics research, and suggest that three of the four derived basis functions resemble three basis functions typically seen in models of standard normal trichromat color vision. Moreover, for CA and LG participants a fourth derived basis function was found to agree with information from both participants' opsin genotypes, and accords with participant's predicted photopigment opsin phenotypes — that is, for both spectral space and color space analyses, the fourth derived basis exhibits peak response sensitivity occurring between basis response functions are presumed to track normal medium- and long-wavelength sensitive response curves. Such a result is surprising, but would be expected if in a potential tetrachromat a fourth class of L-cone photopigment is phenotypically expressed and is contributing to the visual processing of color information.

By comparison, results from 4 basis function analyses of MacBeth Color Checker (MCC) data produced variable findings (Fig. 8e & Fig. 9e ). That is, optimization procedures constrained to identify four basis functions for MCC data identified discordant solutions across spectral space and color space analyses, implying that while three bases derived in MCC solutions resembled those from a standard trichromat model (Fig. 6c & 6d), ambiguity was seen for the determination and interpretation of the fourth basis for MCC optimizations when compared across spectral and color space. In conjunction with our other quantitative measures we interpret this to imply that MCC reflectance measure data was optimized by three basis function solutions, but, unlike the data of CA and LG, was not uniformly optimized across the four basis function analyses. Note that compared to recent results examining much larger stimulus sets (involving 1269 Munsell Color samples [33] and

hyperspectral natural scene data [34]) our results can be interpreted as consistent with findings suggesting that five or more basis functions are needed for trichromat processing of color information when more extensive datasets are considered. Thus, in view of existing literature, the present basis function findings for the MCC, and for our potential tetrachromat data sets, provide confidence that our optimization procedure is tracking color processing functions represented in those data, and can be explored as an analytic tool for use in more extensive studies of color process modeling.

To augment our basis function investigations we also extended methods to conduct analyses examining dimensionality of the three reflectance spectra datasets. We employed well-known ISOMAP procedures to explore the best low-dimensional embeddings of the high-dimensional data sets, and we derived the data dimensions inherent in the three sets of spectra (MCC, CA, and LG). Our dimensionality results find that spectra from the MCC (which is a color calibration stimulus based on a trichromat standard observer model) was best fit by a three-dimensional color appearance model. This result was expected and provides further confidence in our methods because it suggests that the MCC reflectance spectra dimensionality is well modeled by a trichromat model.

In comparison to the ISOMAP results for the MCC, ISOMAP dimensionality observed for empirically reproduced spectra data sets varied across our two potential tetrachromat participants. That is, we found spectral measurement data from one potential tetrachromat individual (LG) were best fit by a three-dimensional model, similar to the observed MCC dimensionality result. Whereas, the measured spectra from our second potential tetrachromat (CA) was best fit by a four-dimensional model.

These results suggests that although LG has a potential tetrachromat genotype, LG's color appearance data is best captured by only three dimensions in ISOMAP analyses, whereas CA's data is best modeled by four dimensions of color experience using ISOMAP. While such a finding may seem ambiguous, it is perhaps not surprising since it is known that not every observer with the genetic potential for four distinct classes of retinal photopigments will ultimately realize four dimensional color processing [19]. Thus, although further work, using more extensive stimulus spaces, is needed to tease apart the details of the present findings with respect to potential tetrachromat processing, in light of other results [16, 17], the present dimensionality findings suggest that the empirical and analytic procedures described here might be sensitive enough to pick up perceptual learning effects that possibly enhance potential tetrachromat color processing.

Future investigations aim to investigate more extensive sets of color stimuli to improve the ecological-validity of the color reproduction task, and to develop a four-channel color processing system with spectral characteristics similar to tetrachromatic basis functions.

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